Berry Curvature in Optical Systems

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Ref

Textbook

A. Yariv and P. Yeh, Photonics: Optical Electronics in Modern Communications (Oxford University Press, New York, 2007)

Theory

Miri, M. A., Regensburger, A., Peschel, U., & Christodoulides, D. N. (2012). Optical mesh lattices with PT symmetry. Physical Review A - Atomic, Molecular, and Optical Physics, 86(2), 112. https://doi.org/10.1103/PhysRevA.86.023807

Experiment

Wimmer, M., Price, H. M., Carusotto, I., & Peschel, U. (2016). Experimental Measurement of the Berry Curvature from Anomalous Transport, 13(February). https://doi.org/10.1038/nphys4050

a Brief Review of Berry Curvature

$$A^n_{\mu,\lambda} = i\partial_\mu \langle n| \partial_\lambda |n\rangle - \mu \leftrightarrow \lambda \tag{1}$$

The BC can be defined in any wave equation.

the Dynamic Effects of BC

Generally:

$$\langle\psi(t)| -\partial_\mu H |\psi(t)\rangle = \langle 0| -\partial_\mu H |0\rangle - A^n_{\mu,\lambda}\frac{d\lambda}{dt} + O(\frac{d\lambda}{dt}^2)$$

Last time, we introduced "anomalous velocity" :

$$v^n = v^n_g - k\frac{dv^n}{dk} \approx v^n_g + \frac{\partial\mu}{\partial t}A^n_{\mu,\lambda}$$

Artificial Gauge Field In Optical System:From PRA 86.023807

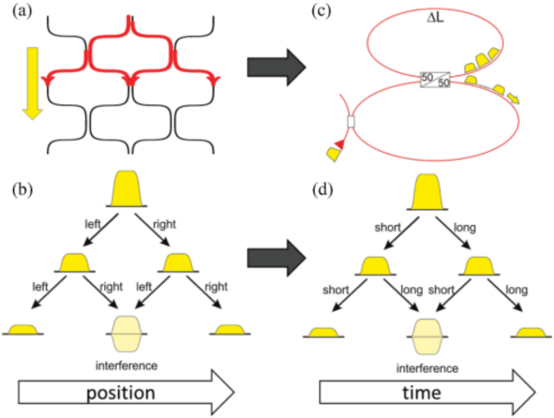

Figure: Map

Artificial Gauge Field In Optical System:From PRA 86.023807

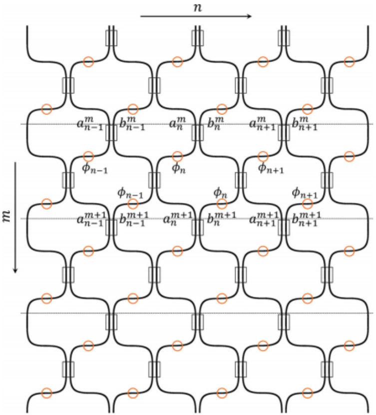

FIG. 2. (Color online) An optical mesh lattice. The lattice is composed of an array of waveguides, which are periodically coupled together in discrete intervals. Circles indicate the position of phase elements and rectangles indicate the coupling regions. The dashed lines show the discrete points where the field intensity is evaluated before coupling occurs.

Artificial Gauge Field In Optical System:nphys4050

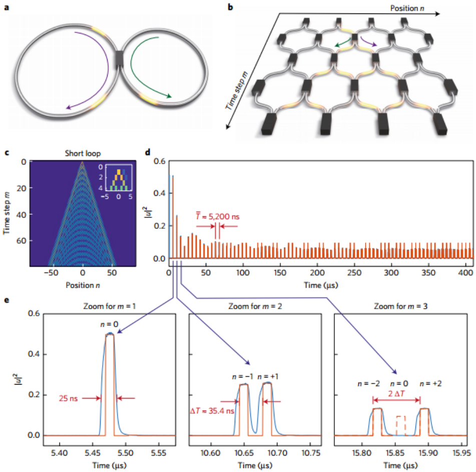

Figure:

$$\begin{pmatrix} u_n^{m+1} \\ v_{n+2}^{m+1} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\phi(m)} & ie^{i\phi(m)} \\ 1 & i \end{pmatrix} \begin{pmatrix} u_{n+1}^{m} \\ v_{n+1}^{m} \end{pmatrix}$$

By inserting Bloch Theorem:

$$\begin{pmatrix} u_n^{m} \\ v_n^{m} \end{pmatrix} = \psi_j e^{-i\frac{\theta m}{2}} e^{-i\frac{Qn}{2}}$$

With $\phi(m) = 0$,we obtain the dispersion relation:

$$\cos\theta = \frac{1}{2}(cos\,(Q) - 1)$$

Setting $\phi\,(m) = \begin{cases} \varphi, \text{odd } m \\ -\varphi, even\ m \end{cases}$,the dispersion relation turns out to be:

$$\cos\theta = \frac{1}{2}(cos\,(Q) - cos\varphi)$$

Band Struction

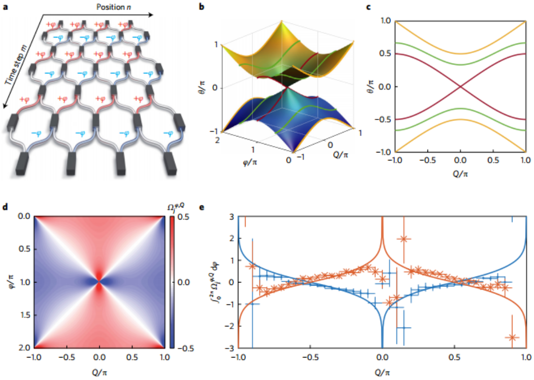

Figure:

Measurement of BC

To get geometric effect, it's required to have a changing ϕ,so:

$$\phi\left(M\left(m\right)\right)=\begin{cases}-\varphi\left(M\left(m\right)\right), \text{odd m}\\ \varphi\left(M\left(m\right)\right), \text{even m}\end{cases}, \varphi=\varphi_0 M\left(m\right), M\left(m\right)=\left\lfloor\frac{m}{2}\right\rfloor$$

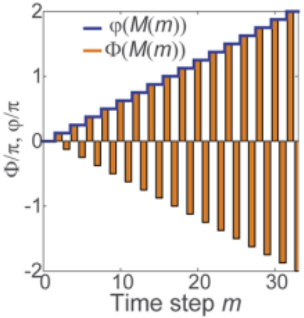

Figure:

Measurement of BC

Then,the COM of the Gaussian envelope evolves with a velocity:

$$v_j\left(M\right)=v_j^g\left(\varphi\left(M\right),Q\right)+\frac{\partial\varphi\left(M\right)}{\partial M}A_j^{\varphi(M),Q}$$
$$=\frac{\partial\theta_j\left(\varphi\left(M\right),Q\right)}{\partial Q}+\varphi_0 A_j^{\varphi(M),Q}$$

the corresponding observable quantity can be the shift in COM of Gaussian envelope along the axis n as a function of M:

$$n_j^{\varphi_0}\left(M\right)=\int_0^M\left[\frac{\partial\theta_j\left(\varphi\left(M'\right),Q\right)}{\partial Q}+\varphi_0 A_j^{\varphi(M'),Q}\right]dM'$$

To remove the effect of dynamic effect, compare the propagation under a reversal of the sign of the phase manitude:

$$\Delta n_j\left(M\right)=n_j^{\varphi_0}\left(M\right)-n_j^{-\varphi_0}\left(M\right)=2\varphi_0\int_0^M A_j^{\varphi(M'),Q}dM'$$

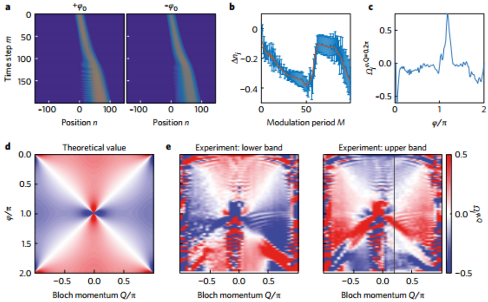
a
$+\varphi_0$
$-\varphi_0$
Time step m
0
50
100
150
-100
0
100
Position n
b
0
-0.2
-0.4
50
Modulation period M
c
0.5
0.0
-0.5
1
2
φ/π
d
Theoretical value
1.0
1.5
2.0
Bloch momentum Q/π
e
Experiment: lower band
Experiment: upper band

Figure: