Understanding Circuits in Quantum Optics way: Quantum Simulations, Computation and Relative Techniques

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Overview

1. Introduction to Quantum Circuit Devices
   - Scheme and Device
   - Qubit engineering
   - Interaction and Controlling
   - Dissipation and Dephasing
2. Relative Theoretical Techniques
   - Effective Hamiltonian Method
   - From Rabi Model to JC Model
   - Dressed State Picture
3. Promising Application and Outlook
   - Quantum information processing on SQCs
   - Quantum Simulations in circuitQED
4. Open Question
5. References and Supplematenry Codes

Overview

Introduction to Quantum Circuits

The interaction between EM fields and matter and the matter-matter interaction via EM field present so many interesting phenomenon and promising applications. They are firstly explored in a system named *cavity quantum electrodynamics*.

Although most atomic physics and quantum optics are focused on phenomena in the optical domain, the last two decades have witnessed a greatly increased interest in superconducting quantum circuits(SQCs).

As quantum information theory finally enter the vision of quantum optics physicist, using SQCs to process quantum information, investigate quantum properties in many body phenomenon and form so-called quantum devices has come to the fore.[WYH+16, HLC+17, GFML, NSO+10, GAN14]

Overview

Scheme

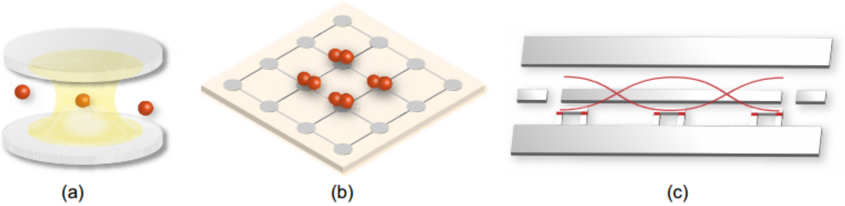

Figure: Systems from optical domain to more scalable and controllable quantum systems. (a) cavityQED with several *free* particle. (b)Ion trap with several interacting ions in a well. (c)Superconducting quantum circuits coupled by Transmission line.

Device: SC-Insulator-SC Josephson junction.

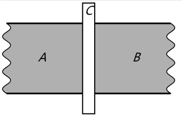

Figure: Structure of Josephson junction

Properties:

- Macroscopic systems behave quantum mechanically.
- Intrinsic capacitance C_J;Supercurrent:$I = I_c \sin\phi, \hbar\frac{\partial\phi}{\partial t} = 2eV$,;Energy:$E = -E_J \cos\phi, E_J = \frac{\hbar I_c}{2e}$
- Non-linearity which bring non-equally spaced energy spectrum to SQCs by anharmonicity.
- Microwave regime.
- long dephasing time.

Example 1:Hamiltonian approach

Degree of freedom in circuit:

$$\Phi_n(t) = \int_{-\infty}^{t} V_n(t')\,dt'$$

$$Q_n(t) = \int_{-\infty}^{t} I_n(t')\,dt'$$

Constraint condition: Kirchhoff's laws.
Note that kinetic part and potential part is contributed by capacitance and inductance,respectively:

$$\mathcal{L}_C = \frac{CV^2}{2} = \frac{C\left(\dot{\Phi}_1 - \dot{\Phi}_2\right)}{2}, \mathcal{L}_I = -\frac{LI^2}{2} = \frac{\left(\Phi_1 - \Phi_2\right)^2}{2L}\left(V = L\dot{I}\right)$$

Example 1:Hamiltonian approach

For Josephson junction:

$$I_J = I_c \sin\phi$$

$$\hbar\dot{\phi} = 2eV(t), \phi = \frac{2e(\Phi_1 - \Phi_2)}{\hbar}$$

From these equations, we obtain:

$$L_I = -\int_{-\infty}^{t} I(t')V(t')dt' = -E_J(1-\cos\phi)$$

Then, the Lagragian and Hamiltonian of the whole system is:

$$L = \frac{C_J\left(\dot{\Phi}_1 - \dot{\Phi}_2\right)^2}{2} - E_J(1-\cos\phi) = \frac{C_J\hbar^2}{8e^2}\dot{\phi}^2 - E_J(1-\cos\phi)$$

$$H = \frac{2e^2}{C_J\hbar^2}\dot{\phi} + E_J(1-\cos\phi), \pi_\phi = \frac{C_J\hbar^2}{4e^2}\dot{\phi}$$

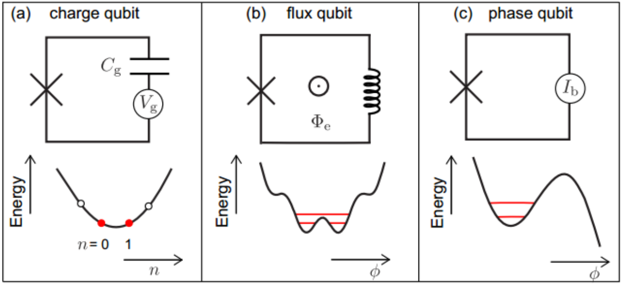

Figure 2: Schematic diagrams of the three basic superconducting quantum circuits and their potential energies. (a) Charge qubit, (b) flux qubit, and (c) phase qubit.

Figure: Three types of basic SQCs.(a)charge-qubit circuit with $E_c/E_J \approx 10.$. Condition: $\Delta E \gg E_C \gg E_J \gg k_B T$. (b)Flux qubit circuits. Condition: Three junctions: $E_J \gg E_C \gg k_B T, 0.6 < \alpha < 0.7$. (c)phase qubit circuits.Condition $E_J/E_c \gg 1$

Example 2:charge qubit circuit

For charge qubit circuit, the Hamiltonian can be readily written as :

$$H = 4E_C(n - n_g)^2 - E_j \cos\phi$$

Choose n and ϕ as a pair of quantum mechanical conjugate operators and represent Hamiltonian in the charge basis $|n\rangle$:

$$H = \sum_n \left[4E_C(n - n_g)^2 |n\rangle \langle n| - \frac{1}{2} E_J \left(|n+1\rangle \langle n| + h.c. \right) \right]$$

Overview

Qubit from anharmonicity

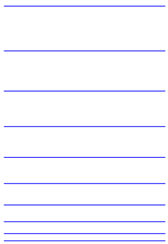

Figure: Anharmonicity make it possible to choose a two-state system by considering the two lowest-energy levels, e.g : $H_q = -2E_C(1-2n_g)\sigma_z - E_J\sigma_x/2$

Other Scheme

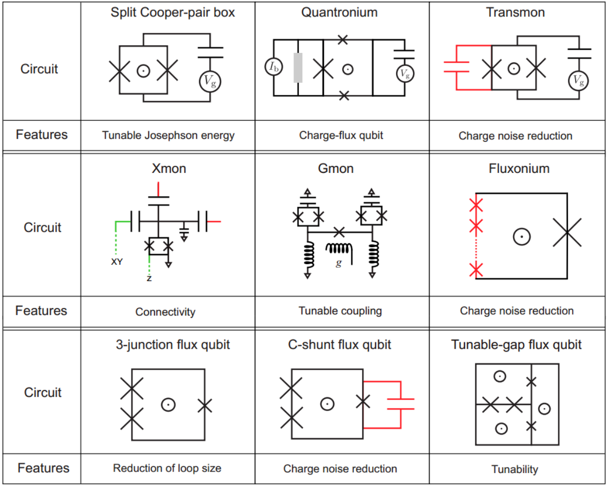

Figure: Other rather complex scheme

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Resonator

In terms of the capacitor charge Q and the inductor current I, the Hamiltonian of the LC oscillator is written as:

$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

It's quadric and can be easily represented in the form:

$$H = \hbar\omega_0 \left(a^\dagger a + \frac{1}{2} \right)$$

Where $a^\dagger$ and a are bosonic creation and annihilation operator. Just like quantized radiation field interacting with natural atoms, the transmission line resonator can be coupled with artificial SQCs atom.

Example 3: Transmon qubits with TLR

When the set of parameters of a Transmon circuit lays in so-called strong-coupling regime, the rotating wave approximation can be applied to this system:

$$H = H_{TLR} + H_Q + H_I$$

$$H_{TLR} + H_Q = \omega_e a^\dagger a + \omega_q b^\dagger b + \frac{\alpha}{2} b^\dagger b^\dagger b b$$

$$H_I = g\left(a^\dagger b + b^\dagger a\right)$$

Properties:

- the Hamiltonian is J-C like but transmon qubit have anharmonicity α
- w_e and w_q are controllable, which can be use to construct a fully controllable qubit.

Example 3: Transmon qubits with TLR

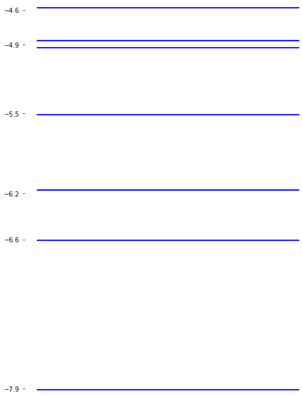


Figure: Energy level of dressed state of Transmon qubits coupled to a TLR. The subspace $\{|4\rangle = |g,1\rangle, |5\rangle = |f,0\rangle\}$ can be treated as an effective and controllable qubit.

General Quantum Control

In quantum control we look to prepare some specific states, effect some state-to-state transfer, or effect some transformation (or gate) on a quantum system. We are going to solve the problem: given a specific quantum system with known time-independent dynamics generator and set of externally controllable fields for which the interaction can be described by control dynamics generators:

- What quantum gate can we achieve?
- What is the shape of the pulse required to achieve this?
- How to optimize the fidelity of the quantum gate?
- How to get rid of classical noise due to pulse and quantum noise due to dephasing(crosstalk, coupled to environment)?

Example 4:Quantum Control on Transmon qubits

Introduce a term describing drive field on the artificial atom, the full Hamiltonian can be written as:

$$H = H_{TLR} + H_Q + H_I + H_D$$
$$H_D = \frac{\Omega\left(t\right)}{2}\left(e^{i\phi}b + b^{\dagger}e^{-i\phi}\right)$$

To achieve the goal gate $\pi/2$ SWAP gate, we can apply such a pulse sequence:

$$g_{eff} = \begin{cases} g_{\max}\sin^2\dfrac{\pi t}{2\Delta t}, t \leqslant \Delta t \\ g_{\max}, \Delta t < t < T - \Delta t \\ g_{\max}\sin^2\dfrac{\pi\left(T-t\right)}{2\Delta t}, T - \Delta t \leqslant t \end{cases}$$

Example 4:Quantum Control on Transmon qubits

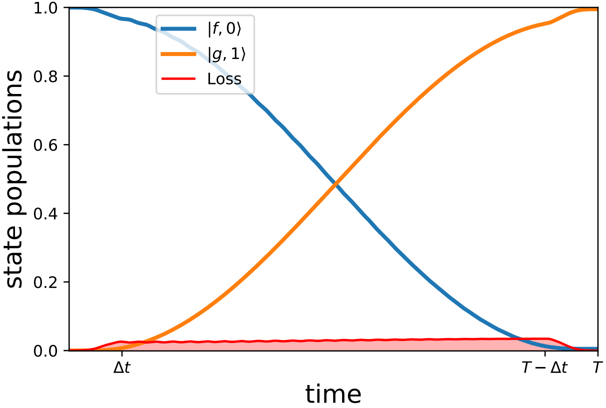


Figure: Quantum control of transmon qubit. By using holonomic quantum computation technique, the fidelity reach 99.999% and become local-noise silent.

Example 4:Quantum Control on Transmon qubits

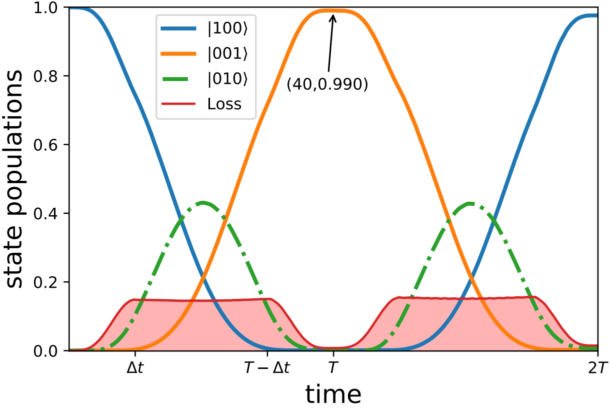


Figure: Implementation of SWAP gate of a TLR coupled to two qubits.

Symmetry and selection rules

Qubits	Special values	Inversion symmetry	Selection rules
Charge qubits	$n_g = 0.5$	Yes	Yes
Charge qubits	$n_g \neq 0.5$	No	No
Flux qubits	$f = 0.5$	Yes	Yes
Flux qubits	$f \neq 0.5$	No	No
Phase qubits	—	No	No

Table 2: Summary of selection rules and symmetries of potential energies for SQCs implementing charge qubits at the gate-charge numbers $n_g = 0.5$ and $n_g \neq 0.5$, flux qubits at the reduced magnetic fluxes $f = 0.5$ and $f \neq 0.5$, and phase qubits.

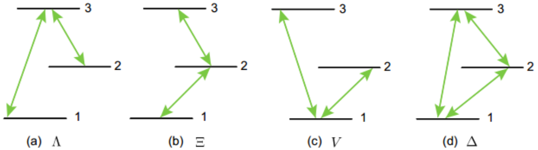


Figure 4: Schematic diagrams for different transition configurations of three-level atoms. (a) Λ-type transitions. (b) Ladder-type transitions, also known as cascade-type, Ξ, or Σ transitions. (c) V-type transitions. (d) Δ-type transitions, which do not occur in natural atoms, but are possible in some superconducting artificial atoms.

Overview

Linblad master equation

Dissipation and Dephasing happen due to crosstalk between the qubit subspace and others and the coupling with environment.
By considering the conditions below, a phenomenological equation can be obtain to include dissipation due to environment:

- Separability. At $t = 0$ there are no correlations between the system and its environment such that the total density matrix can be written as:$\rho_{tot}(0) = \rho(0) \otimes \rho_{env}(0)$
- Born approximation. The state of environment after interaction does not significantly change. The system and the environment remain separable throughout the evolution.
- Markov approximation. Short-memory environment.
- Secular approximation(not necessarily required).

Linblad master equation

$$\dot{\rho}(t) = -\frac{i}{\hbar}\left[H(t), \rho(t)\right] + \sum_{n} \frac{1}{2}\left[2C_n \rho(t) C_n^{+} - \left\{\rho(t), C_n^{+} C_n\right\}\right]$$

Where ρ is the density operator of the system we consider, C_n is the set of collapse operator including all kinds of quantum jumps. Note that the evolution of the density operator is no longer unitary.
Another part of loss is that the qubit(or system) is a virtual subspace of a,e.g, infinite-level system. The crosstalk and quantum jumps from the subspace to outer space will bring loss of fidelity and effective loss. We need some powerful weapons to obtain a non-unitary effective theory. We may turn to effective field theory for a hand.

Summary 1. Other Topics

Other topics I don't mention:

- Read out, reconstruction and representation of a quantum state.
- Other components, such as:Beam-splitter, circulators, switches, routers, mixers, etc.
- Coupling strength.
- Quantum information(e.g entanglement entropy) spreading in the circuits

	Weak coupling	**Strong coupling**	**USC**	**DSC**
Coupling strength	$g < \max\{\gamma, \kappa\}$	$g > \max\{\gamma, \kappa\}$	$g \gtrsim 0.1\omega_{\mathrm{r/q}}$	$g \gtrsim \omega_{\mathrm{r/q}}$
Model	JC	JC	Q. Rabi	Q. Rabi

Overview

Overview

Perturbation Theory-Textbook Version

First, we have a diagonalizable Hamiltonian H_0

$$H_0 |i\rangle = \varepsilon_i |i\rangle$$

Then,

$$H_0 |a\rangle = \varepsilon |a\rangle$$
$$H_0 |\alpha\rangle = \varepsilon_\alpha |\alpha\rangle$$

We're going to solve

$$(H_0 - \varepsilon) |A\rangle = \left(\Delta - H'\right) |A\rangle \tag{1}$$

By defining the projector below:

$$\begin{aligned} P &= \sum_a |a\rangle \langle a| \\ Q &= 1 - P = \sum_\alpha |\alpha\rangle \langle\alpha| \\ K &= \sum_\alpha \frac{|\alpha\rangle \langle\alpha|}{\varepsilon_\alpha - \varepsilon} \end{aligned} \tag{2}$$

Perturbation Theory-Textbook Version

We can obtain:

$$|A\rangle = LP\,|A\rangle$$
$$L = \sum_n L_n$$

Where

$$L_0 = P, L_1 = -KH'P; L_n = K\left(-HL_{n-1} + \sum_{i=1}^{n} L_i H' L_{n-i-1}\right)$$

And the effective evolution equation:

$$PH'LP\,|A\rangle = \Delta P\,|A\rangle$$
$$\sim \tilde{H}|A\rangle_k = \Delta|A\rangle_k$$
$$\tilde{H} = PH'L = PH'P - PH'KH'P + ...$$

Perturbation Theory-Field Theory Version

The spectrum of the differential equation can be obtained from the resolvent or Green function:

$$\begin{aligned} &G(z) = \frac{1}{z-H}, G_0(z) = \frac{1}{z-H_0} \\ &G - G_0 = G_0 (H - H_0) G \\ &G = G_0 (I + H_I G) \\ &G = G_0 + G_0 H_I G_0 + G_0 H_I G_0 H_I G_0 + ... \\ &= G^0 + G^1 + G^2 + ... \end{aligned}$$

From Dyson's equation, the physical meaning is basically that all the interaction(scattering) information are encoded in the perturbation Hamiltonian H_I.

Our task is to find another operator Σ defined in the subspace but resemble the operator H_I

Perturbation Theory-Field Theory Version

By applying projector, one can find:

$$\Sigma_{kl}\left(z\right) = H^I_{kl} + \sum_{m \neq ij} H^I_{km} \frac{1}{z - E_m} H^I_{ml} + ...$$

Hubbard-Stratonoviach Transformation

This method is basically to decouple the interaction term and serves as a powerful tools in mean-field theory:
When there is interaction term,e.g $\exp\{-\psi_m V_{mn}\psi_n\}$ Then, the term can be decouple by introducing a new set of fields(or parameter)ϕ:

$$\exp\{-\psi_m V_{mn}\psi_n\}$$
$$= \int D\phi \exp\left\{-\frac{1}{4}\phi_m V^{-1}_{mn}\phi_n - i\phi_m\psi_m\right\}$$

Applying saddle-point approximation can then help to construct one kind of effective theory named "mean field theory".

Schrieffer-Wolff transformation-Canonical Version

The basic idea of Schrieffer-Wolff transformation is to obtain the effective field theory by Canonical Transformation:

$$
\begin{aligned}
& H = H_0 + H_I \\
& [H_0, S] = -H_I \\
& e^S H e^{-S} = H_0 + [S, H_I] + \frac{1}{2!}[S, [S, H]] + \ldots \\
& = H_0 + H_I + [S, H_I] - H_I + \frac{1}{2}[S, -H_I] + \ldots \\
& = H_0 + \frac{1}{2}[S, H_I] + \ldots
\end{aligned}
$$

Several terms can lead to a good low-energy approximation of the original model.

Schrieffer-Wolff transformation-Path Integral Version

However, HS-transformation and simple saddle point analysis can also lead to low-energy theory, which is more eligible in my mind.

Holstein-Primakoff transformation

Spin operators $\hat{S}^{\pm}, \hat{S}$ are specified in terms of bosonic creation and annihilation operator $b^{\dagger}$ and b:

$$S_i^- = b_i^{\dagger}\sqrt{2S - b_i^{\dagger}b_i}$$
$$S_i^+ = \sqrt{2S - b_i^{\dagger}b_i}b_i$$
$$S_i^z = S - b_i^{\dagger}b_i$$

In the large-N situation:

$$S_i^- = b_i^{\dagger}\sqrt{2S - b_i^{\dagger}b_i} \sim \sqrt{2S}b_i^{\dagger}$$
$$S_i^+ = \sqrt{2S - b_i^{\dagger}b_i}b_i \sim \sqrt{2S}b_i$$
$$S_i^z = S - b_i^{\dagger}b_i$$

Another useful representation of quantum mechanical spin is Schwinger boson representation: $S^+ = a^{\dagger}b, S^z = \frac{1}{2}\left(a^{\dagger}a - b^{\dagger}b\right)$

Jordan-Wigner Transformation

A representation for spin-1/2 can be obtained in terms of Fermion operators:

$$S_i^- = f_i^+ e^{i\pi \sum\limits_{j<i} f_i^+ f_i}$$

$$S_i^+ = e^{-i\pi \sum\limits_{j<i} f_i^+ f_i} f_i$$

$$S_i^z = f_i^+ f - \frac{1}{2}$$

Where $f_i^\dagger$andf_i is the fermion creation and annihilation operator.

Example 5. Low-energy model of J-C model

We begin form the Jaynes-Cummings Hamiltonian :

$$
\begin{aligned}
H &= H_0 + H_I \\
H_0 &= \omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z \\
H_I &= g\left(a\sigma_+ + \sigma_- a^\dagger\right)
\end{aligned}
$$

Applying S-W transformation, one can obtain:

$$
H = \left(\omega_r + \Xi\sigma_z\right) a^\dagger a + \frac{\omega_q + \Xi}{2}\sigma_z
$$

Where

$$
\Xi = \frac{g^2}{\omega_q - \omega_r}, \left|g/\left(\omega_q - \omega_r\right)\right| \ll 1
$$

Overview

From Rabi Model to JC Model

Quantum Rabi Model:
he most fundamental setups of Quantum optics(cavity quantum electrodynamics) can be described by the general quantum Rabi model:

$$H_{Rabi} = \omega_r(a^\dagger a + \frac{1}{2}) + \frac{\omega_q}{2}\sigma_z + g(a^\dagger + a)(\sigma_+ + \sigma_-)$$

where $2g$ denotes an experimental parameter the *vacuum Rabi frequency*. $a^\dagger$and a are bosonic creation and annihilation operator, respectively. σ_+,σ_- are ladder operators in the σ_z representation and $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$
The Hamiltonian defined above can describe a two-level atom coupled to a standing radiation field in the F-P cavity. The model also becomes a fundamental one in circuit quantum electrodyniamcs to describe the qubit coupled to a transmission line.

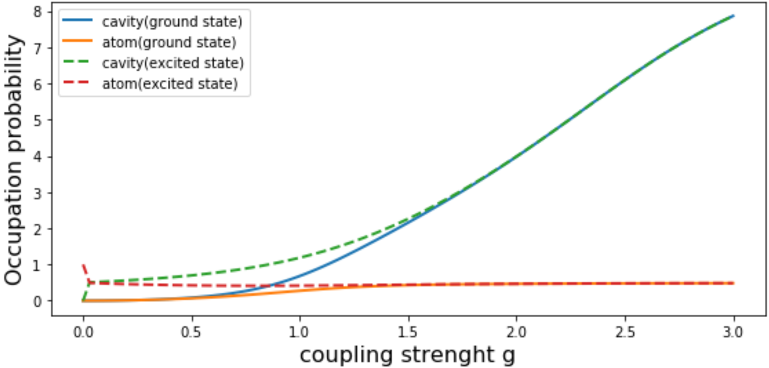
From Rabi Model to JC Model

hen, the physical picture of this model can be:
- the self energy of the atom and the radiation field; - the light-matter interaction part which contains four part:$\sigma_+ a$,$\sigma_- a^\dagger$,$\sigma_- a$,$\sigma_+ a^\dagger$.
The first term and the second term in the interaction part describe the procedure where the atom raises to excited state by absorbing a photon or drop to the ground state by emitting a photon. The third and fourth term can be a little confusing but it really comes due to *uncertainty relation*. Now, let's do some reasonable simplifications and add some properties to the spherical chicken model. First, due to experimental consideration, the interaction strength in cavity lay in the region:$(\omega_q + \omega_r) \gg g, |\omega_q - \omega_r|$ then after *rotating wave approximation* we can obtain so-called Jaynes-Cummings model:

$$H_{JC} = \omega_r(a^\dagger a + \frac{1}{2}) + \frac{\omega_q}{2}\sigma_z + g(a^\dagger \sigma_- + a\sigma_+)$$

As the cavity has finite volume, we sometimes treat the $\frac{1}{2}$ term as a bias and ignore it.

From Rabi Model to JC Model



Overview

Dressed State Picture

Now we solve the Jaynes-Cumming model analytically: The J-C Hamiltonian H_{JC} can be diagonalized in the subsapce $\{|g\rangle|n+1\rangle, |e\rangle\}$ with the eigenvalues:

$$E_n^{\pm} = E(|\pm, n\rangle) = \omega_r(n+\frac{1}{2}) \pm \frac{1}{2}\sqrt{\Delta^2 + \Omega_{n,r}^2}$$

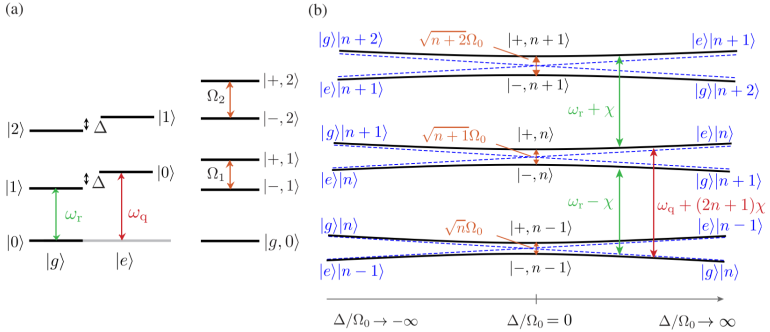
and the corresponding eigenstates:

$$|+, n\rangle = \cos(\frac{\theta_n}{2})|e\rangle|n\rangle + \sin(\frac{\theta_n}{2})|g\rangle|n+1\rangle$$

$$|-, n\rangle = -\sin(\frac{\theta_n}{2})|e\rangle|n\rangle + \cos(\frac{\theta_n}{2})|g\rangle|n+1\rangle$$

Where $\Delta = \omega_q - \omega_r$, $\Omega_n = \sqrt{\Delta^2 + (n+1)\Omega_0^2}$,$\Omega_0 = 2g$
The effective coupling between different ladders is $\omega_r + \chi$ and $\chi = \frac{g^2}{\Delta}$

Dressed State Picture



Example6: spectrum method to determine coupling strength

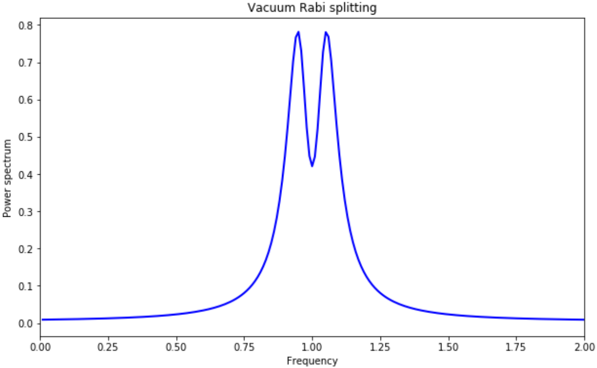


Figure: Vaccum Rabi Splitting:$S(\omega) = \int_{-\infty}^{\infty} \left\langle a^\dagger(\tau)a(0)\right\rangle e^{-i\omega\tau} d\tau$

Example6: spectrum method to determine coupling strength

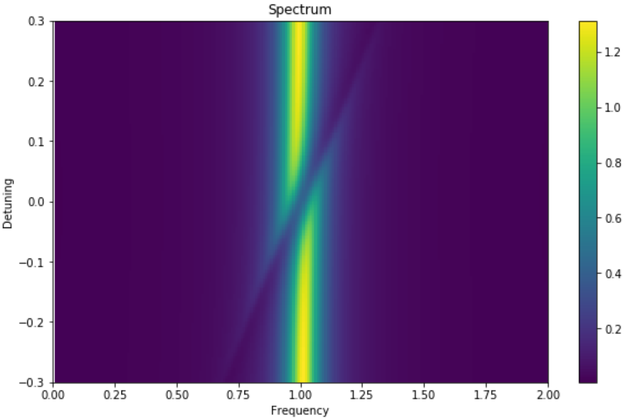


Figure: Anti-crossing in spectrum-detuning diagram.

Overview

Overview

DiVincenzo Criteria for QIP

- Scalable qubits.
- Initializaiton.
- Long coherence times.
- Measurement.
- Universal gates.
- error correction.
- quantum memory.

Overview

Quantum Simulations

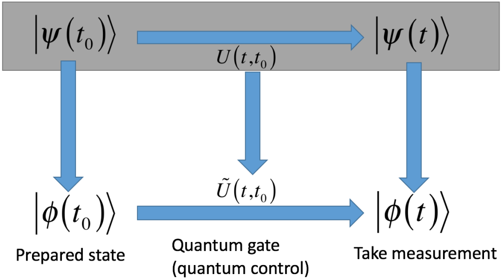


Figure: Scheme of quantum simulations. The original system in the gray block is less accessible and controllable. A well-defined simulator means that its initial state can be prepared, the evolution of the state can be engineered, and the final state can be measured. After mapping from the original system to the simulator and performing quantum gates , one can extract information from simulator. For the two system obey the same physical law, the information one obtained can be remapped to original system.

Example7 Simulating Topological Insulators

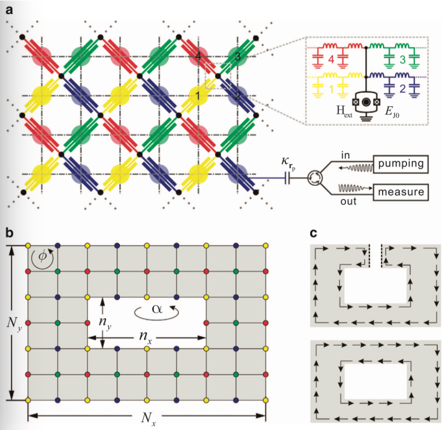


Figure: Implementation of edge states by SQCs. The Hamiltonian can be readily written as :$H = \sum_i \omega_i a_i^\dagger a_i + \left(\sum_{\langle i,j \rangle} e^{-i\phi} a_i^\dagger a_j + h.c. \right) + (P a_k e^{i\Omega t} + h.c.)$

Example7 Simulating Topological Insulators

We first point out that the Hamiltonian can be rewritten as below:

$$H = \psi^\dagger \mathcal{H} \psi + A(t)\,\psi + \psi^\dagger A^*(t)$$

Where $\psi = (a_1, a_2, ..., a_N)$ denotes the vector of field operator. Then, the Heisenburg picture of Linblad master equation:

$$\frac{d\psi}{dt} = -i[\psi, H] + \sum_{i,j} b\left(\mathcal{L}_i \psi \mathcal{L}_j^\dagger - \frac{1}{2}\left\{\mathcal{L}_j^\dagger \mathcal{L}_i, \psi\right\}\right)$$

In a more compact format, and take the average over quantum configuration, then:

$$\frac{d\langle\psi\rangle}{dt} = -i\left(\mathcal{H}\langle\psi\rangle + A^*(t)\right) - \frac{1}{2}\mathcal{K}\langle\psi\rangle$$

Then we are proceeding to the steady state:

$$\left(\mathcal{H} - \frac{1}{2} i\mathcal{K}\right)\langle\psi\rangle = -A^*$$

Example7 Simulating Topological Transitions

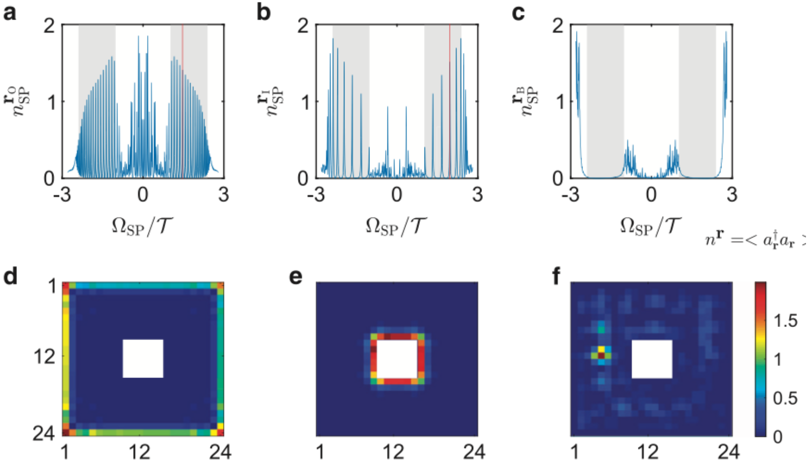


Figure: Simulation result

Example8 Simulating Topological Transitions

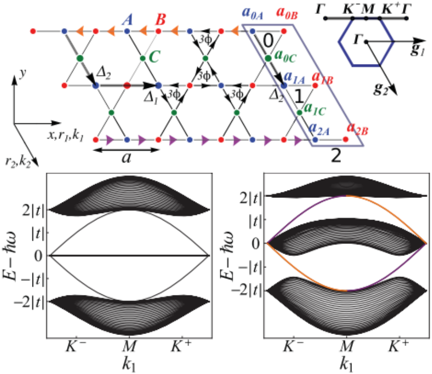


Figure: Kagome Lattice problem. [PHL12]

Example8 Simulating Topological Transitions

Mapping to the Kagome Lattice. In the bosonic representation, the Hamiltonian of can be written as:

$$H\left(\alpha_1, \alpha_2\right) \\ = \psi^{\dagger}\left(\begin{array}{ccc} 0 & e^{i\phi}\cos\left(\alpha_1+\alpha_2\right) & e^{-i\phi}\cos\left(\alpha_1\right) \\ & \omega_1 & e^{i\phi}\cos\left(\alpha_2\right) \\ & & \omega_e \end{array}\right)\psi \tag{3}$$

Where $\psi = \left(\psi_a, \psi_b, \psi_c\right)$ is the vector of bosonic destroy operator. By inspecting the Hamiltonian above and the Hamiltonian of Kagome lattice, we find that they map to each other by a transformation: $\alpha_1 \rightarrow -k_2, \alpha_2 \rightarrow k_3, \alpha_1+\alpha_2 \rightarrow -k_1, H\left(\alpha_1, \alpha_2\right) = -2gH\left(-k_2, k_3\right)$.

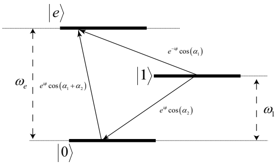


Figure: Scheme of Δ-type atom.

We consider a cycle-three-level atom with arbitary transitions permitted. The atom has two ground states, $|0\rangle$ and $|1\rangle$ and an excited state $|e\rangle$. We begin with this Hamiltonian to demonstrate the essential steps to measure Berry curvature:

$$\begin{aligned} H = {} & \omega_e |e\rangle\langle e| + \omega_1 |1\rangle\langle 1| \\ & + e^{i\phi}\cos(\alpha_1+\alpha_2)|e\rangle\langle 0| + e^{-i\phi}\cos(\alpha_1)|e\rangle\langle 1| \\ & + e^{i\phi}\cos(\alpha_2)|0\rangle\langle 1| + h.c. \end{aligned} \tag{4}$$

Example8 Simulating Topological Transitions

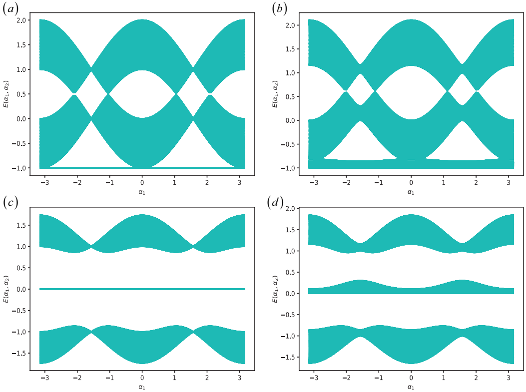


Figure: Implementation of topological transition by SQCs[XCN10]

Example8 Simulating Topological Transitions

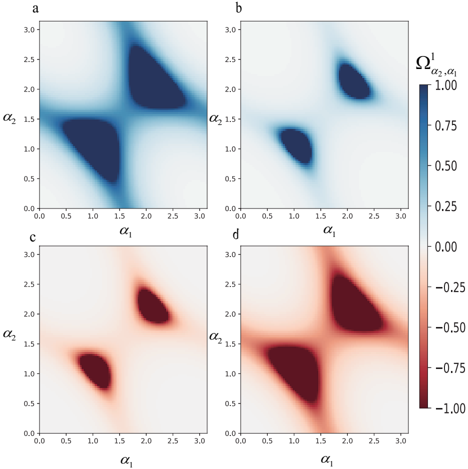


Figure: Simulation result

Physics beyond ordinary condensed matter

This concept was first adopted by Ultracold atoms in optical lattice for the components of the lattice can be fermions or bosons or both of them. So optical lattice can present different statistical properties of a same model Hamiltonian, for instance:

Multicompenents Bose-Hubbard model:

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left(b^{\dagger}_{\sigma i} b_{\sigma j} + h.c. \right) + \frac{U_0}{2} \sum_i n_i \left(n_i - 1 \right) + \frac{U_1}{2} \sum_i \left(S_i^2 - 2n_i \right) - \mu$$

Spin-Orbit coupling Bose-Hubbard model:

$$H = -t \sum_{\langle i,j \rangle} a^{\dagger}_{i\sigma} R_{ij}^{\sigma\sigma'} a_{j\sigma} + \sum_i \left(\frac{U}{2} \sum_{\sigma} \left[n_{i\sigma} \left(n_{i\sigma} - 1 \right) \right] + U_2 n_{i\uparrow} n_{i\downarrow} \right)$$

Overview

Open Question

- Dissipation can be easily introduced to SQCs. Can we simulate quantum chaos?(Phase synchronization and anti-synchronization)
- Many body localization?
- Quantum resources, quantum discord and quantum decoherence, and etc.
- Non-Hermitian Hamiltonian(PT-symmetry protected topological phase)?

Question1:Quantum Chaos in circuits?

Classical chaos has been well studied by a kind of classical circuit named Chua's circuit.[GVL12]

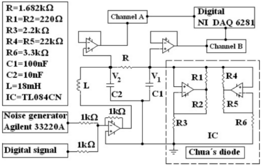
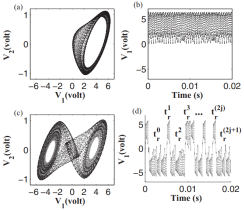


FIG. 1. Experimental setup. A Chua's chaotic circuit driven by superposed periodic signal and white noise.



Question1: Quantum Chaos in circuits?

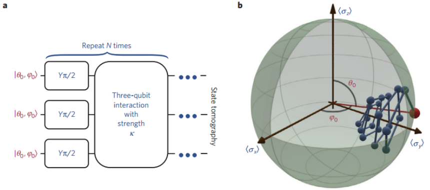
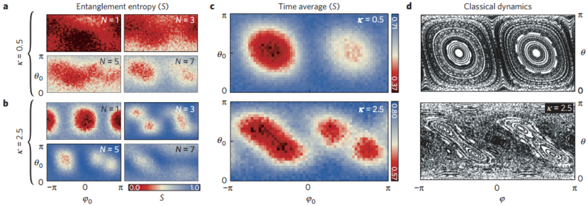


Figure 1 | Pulse sequence and the resulting quantum dynamics. a, Pulse sequence showing first the initial state of the three qubits (equation (4)) followed by the unitary operations for a single time step (equation (3)). These operations are repeated N times before measurement. Single-qubit rotations are generated using shaped microwave pulses in 20 ns; the three-qubit interaction is generated using a tunable coupling circuit controlled using square pulses of length 5 ns for $\kappa = 0.5$ and 25 ns for $\kappa = 2.5$. **b**, The state of a single qubit is measured using state tomography and shown in a Bloch sphere. The initial state is shown in red with subsequent states shown in blue for $N = 1$-20.

Figure: Result from J.Martinis@UCSB&Google



Question2:MBL and Quantum Statistics?

- Basic question: Can conventional quantum statistical mechanics describe long-time properties of any system?
- Eigenstate thermalization hypothesis: If a system at a given temperature does indeed thermalize for every such initial state, then it is quite instructive to consider initializing the system in a pure state that is one of the many-body eigenstates.
- Many-body Anderson Localization violate ETH.

Question2:MBL and Quantum Statistics?

Thermal phase	Single-particle localized	Many-body localized
Memory of initial conditions hidden in global operators at long times	Some memory of local initial conditions preserved in local observables at long times	Some memory of local initial conditions preserved in local observables at long times
Eigenstate thermalization hypothesis (ETH) true	ETH false	ETH false
May have nonzero DC conductivity	Zero DC conductivity	Zero DC conductivity
Continuous local spectrum	Discrete local spectrum	Discrete local spectrum
Eigenstates with volume-law entanglement	Eigenstates with area-law entanglement	Eigenstates with area-law entanglement
Power-law spreading of entanglement from nonentangled initial condition	No spreading of entanglement	Logarithmic spreading of entanglement from nonentangled initial condition
Dephasing and dissipation	No dephasing, no dissipation	Dephasing but no dissipation

Figure: A list of some properties of the MBL phase, SPL phase and thermal phase[NH]

Overview

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All the code is available in my Numerical Lecture of Quantum Optics:
Quantum Optics with Python:https://chaoli.club/index.php/3946
And welcome to discuss in my Zhihu Column:
https://zhuanlan.zhihu.com/QuantumTech
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