

# Hexagonal Cells in Dielectric Liquids Induced by Electric Polarization Pressure

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# Overview

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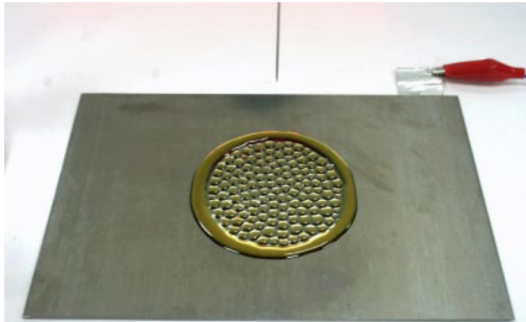
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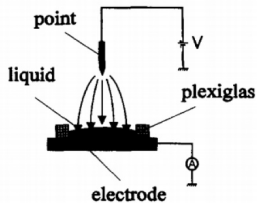
# Introduction

Many typical patterns of hexagonal cells in dielectric have been found for nearly a century, including those driven by electric field, thermal energy, mechanic energy and even chemical potential transport. Corona discharge from a needle is used to induce electrohydrodynamic instability stress in a flat thin layer of dielectric liquid.

In proper voltages, the theory of this phenomenon is presented and verified by our experiment.



# Introduction



Several arguments influence the shape and other parameter of the honeycomb:

- ▶ surface tension coefficient of the liquid
- ▶ thickness of the surface
- ▶ dielectric constant of the liquid
- ▶ field intensity, which is relevant to the voltages and height of the needle tip
- ▶ cleanliness of the liquid

# Deriving Equation

Considering one-dimension situation, the perturbation of the surface can be write as:

$$z = \xi(x, t) = A * \exp(-ikx) \quad (1)$$

The motion of the surface is slight so it's regarded as an ideal fluid:

$$\rho \frac{\partial \phi}{\partial t} + p_t = 0 \quad (2)$$

Where  $\phi$  is the potential function,  $\rho$  density of the fluid and  $p_t$  the total pressure.

$p_t$  is composed of hydrostatic and hydrodynamic comonents,  $p_t = \rho g z - p_e + p_s$  where  $p_e$  is electric press and  $p_s$  the surface tension pressure.

## Deriving Equation

In the presence of an electric field, take the first order approximation of  $p_e$ :

$$p_e = \frac{\varepsilon\varepsilon_0}{2}(E_n^2 - E_\tau^2) \quad (3)$$

The boundary condition  $(\vec{n} \times \vec{E})_{z=\xi} = 0$  implies  $E_\tau = 0$ ; therefore,  $p_e$  gives:

$$p_e = \frac{\varepsilon_0\varepsilon E_n^2}{2} \quad (4)$$

We find that the perturbation only occurs on the surface, so  $\vec{E}$  can be written as:

$$\vec{E} = \vec{E}_0 - \nabla\varphi, \Delta\varphi = 0 \quad (5)$$

$$\vec{E}_n = \vec{E}\vec{n} \quad (6)$$

We take the first-order approximation of the  $p_e$ , which gives:

$$p_e = \frac{\varepsilon_0\varepsilon}{2} \left[ \left( \vec{E}_0 - \nabla\varphi \right)_n \right]^2 = \varepsilon_0\varepsilon E_0 \left( \frac{\partial\varphi}{\partial z} \right)_{z=0} \quad (7)$$

# Deriving Equation

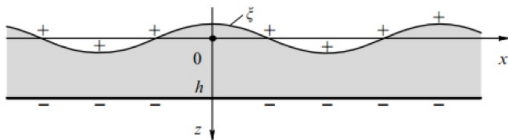
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From Laplace theorem, we can get  $p_s$ :

$$p_s = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)_{z=\xi} \quad (8)$$

Where  $\frac{1}{R_1} = 0$  and  $\left( \frac{1}{R_2} \right)_{z=\xi} = -\frac{\partial^2 \xi}{\partial x^2}$



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# Deriving Equation

The ultimate equation gives:

$$\sigma \frac{\partial^2 \xi}{\partial x^2} - \rho g \xi = \rho \left( \frac{\partial \phi}{\partial t} \right)_{z=0} - \varepsilon_0 \varepsilon E_0 \left( \frac{\partial \varphi}{\partial z} \right)_{z=0} \quad (9)$$

$$\Delta \phi = 0 \quad (10)$$

$$\Delta \varphi = 0 \quad (11)$$

And the boundary condition gives:

$$\varphi(h) = 0$$

$$\phi_z(h) = 0 \quad (12)$$



# Dispersion Relation of the System

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Solutions of the Laplace equations(10) (11)have the form

$$\begin{aligned}\phi &= C_1(t) \cosh [k(z-h)] \exp(-ikx) \\ \varphi &= C_2(t) \sinh [k(z-h)] \exp(-ikx)\end{aligned}\tag{13}$$

In order to determine the system,  $C_2$  have to be detemined. Here boundary conditions are used.

# Dispersion Relation of the System

For electric potential  $\varphi$ , surface boundary gives:

$$\xi_x E_0 = \varphi_x(z=0) \quad (14)$$

It determines  $C_2(t)$ , hence determines  $\varphi$

$$C_2 = \frac{E_0}{\sin kh} \cdot A$$
$$\varphi = \frac{\sinh[k(z-h)]}{\sinh kh} E_0 \xi \quad (15)$$

For the velocity potential, kinematic boundary gives:

$$\phi_z(z=0) = \xi_t \quad (16)$$

# Dispersion Relation of the System

Setting  $C_1(t) = \exp(\alpha t)$ , From (9) we get:

$$\rho g k \sinh kh + \rho \alpha^2 \cosh kh + \sigma k^3 \sinh kh - k^2 \varepsilon \varepsilon_0 E_0^2 \cosh kh = 0 \quad (17)$$

Both sides divide  $\cosh kh$ , the dispersion equation is presented:

$$\rho \alpha^2 = (k^2 \varepsilon \varepsilon_0 E_0^2 \coth kh - \rho g k - \sigma k^3) \tanh(kh) \quad (18)$$

The condition  $\alpha = 0$  determines the system:

$$\varepsilon \varepsilon_0 E_0^2 = (\rho g / k - \sigma k) \tanh(kh) \quad (19)$$

# Experimental Facilities

- ▶ high-speed video cameras capable of recording lightning at up to 2000 images per second.
- ▶ subtly polished needle with spherical tips ( $U_r < 1\%$ )
- ▶ high-voltage power supply (HVPS)
- ▶ .....



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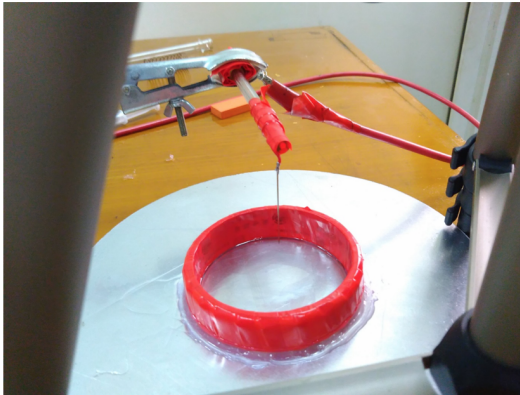
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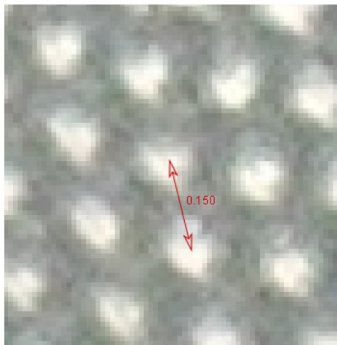
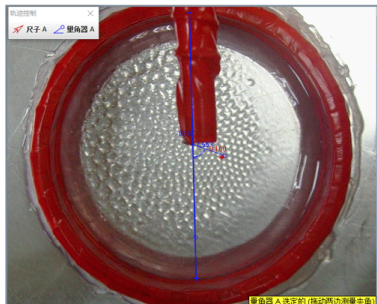
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# Measuring Methods

The video analysis software Tracker is used to measure the length of the wave package.



# Phenomenon

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The phenomenon is very obvious:



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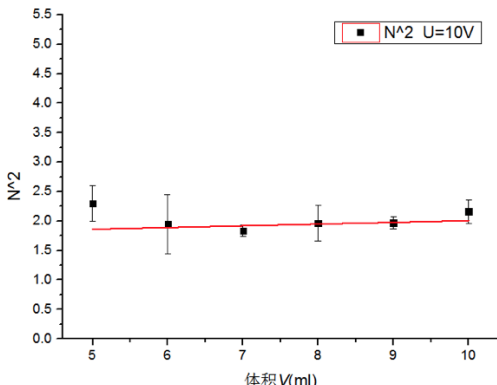
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# Verify Conservative Quantity in Dispersion Relation

Using  $a = \sqrt{\sigma/\rho g}$ ,  $K = ak$ ,  $H = h/a$ , the Dispersion Relation  $\varepsilon\varepsilon_0 E_0^2 = (\rho g/k - \sigma k) \tanh(kh)$  can be written as:

$$N^2 = \frac{\varepsilon\varepsilon_0 E_0^2}{\rho g a} = \left(\frac{1}{K} + K\right) \tanh(KH)$$

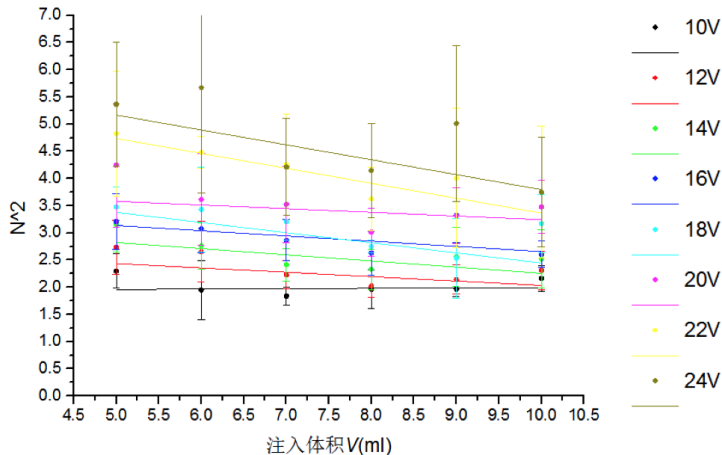
Where  $N^2$  is const





# Verify Conservative Quantity in Dispersion Relation

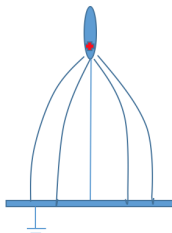
This figure shows  $N^2$  in different voltages. The slope does not equal to zero. We will discuss this error below.



# Verify the Equation

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In this model, the field near the needle can be approximately written:

$$E_0 \approx \frac{2U}{r \cdot \ln\left(\frac{2L}{r}\right)}$$

Where  $r$  is the radius of curvature

By changing the voltage, we can find the  $U^2 - h(orV)$  curve:

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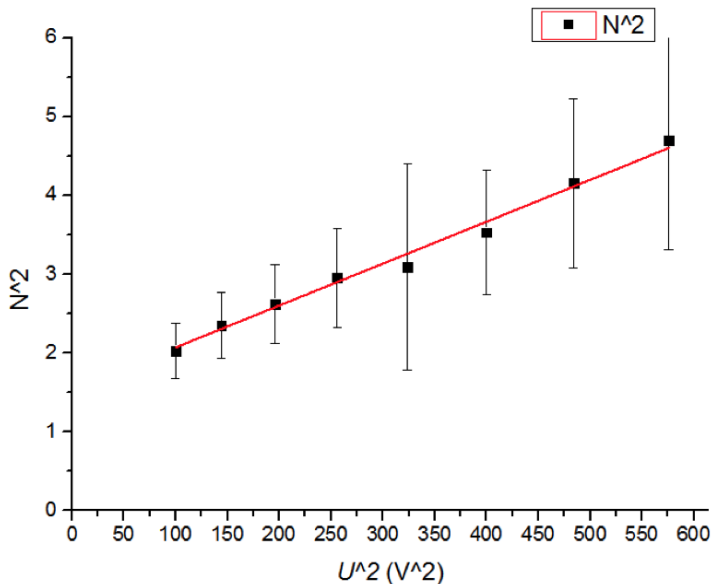
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# Verify the Equation

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In proper voltages, the conservative quantity and the Dispersion Relation is verified in some degree. Some of the data have suggested uncertainty of the measurements. Here comes some discussion about them.

The main error sources of this experiment are:

- ▶ (1) periodic systematic errors from voltage adjustment
- ▶ (2) systematic errors from leakage and loss of injection
- ▶ (3) linear systematic errors aberrations from camera

Item (3) and (1) can be figured out accurately and Item (2) can be reduced as much as possible.

$$\varepsilon\varepsilon_0 E_0^2 = (\rho g/k - \sigma k) \tanh(kh) \quad (19)$$

This equation determines the system generally.

The phenomenon of instability of a dielectric fluid layer in the field of corona discharge is notable for the richness of its physical content(Hydrodynamics,Electrodynamics),the ease of making it available ,and the ability to essentially extend the scope of lecture demonstrations of self-organization.The theory of this phenomenon is tightly linked to traditional courses in general and theoretical physics, and can be studied by solving a series of practical exercises.

- ▶ V V Mayer, E I Varaksina, V A Saranin, Simple lecture demonstrations of instability and self-organization.
- ▶ F. Vega, A.T. Perez, Corona-induced electrohydrodynamic instabilities in low conducting liquids.
- ▶ Albert, P&ez, Electrohydrodynamic Instabilities in Dielectric Liquids Induced by Corona Discharge.
- ▶ V. A. Saranin, V. V. Mayer, and E. I. Varaksina, Instability of Equilibrium of the Liquid Dielectric Surface and Formation of Regular Cellular Structures in the Field of a Corona Discharge

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